Useful Formulas & Constants

Physical Constants

Planck's constant $h = 6.6260755\times10^{-34}$ J·s $= 4.5\times10^{-15}$ eV·s
$= 6.626\times10^{-27}$ erg·s

Dirac's constant $\hbar = h/2\pi = 1.054\times10^{-34}$ J·s
$= 1.054\times10^{-27}$ erg·s

Boltzmann's constant $k_B = 1.380\times10^{-16}$ erg/K
$= 8.62\times10^{-5}$ eV/K $= 1.380\times10^{-23}$ J/K

kT = 25.9 meV at room temperature
= 0.36 meV at liquid-helium temperature (4.2 K)
= 6.7 meV at liquid-nitrogen temperature (4.2 K)

Velocity of light in vacuum $c = 2.99792458\times10^{8}$ m/s

Electron charge $e = 1.602\times10^{-19}$ coulombs

Avogadro number $N_a = 6.0221367\times10^{23}$ particles/mol

Permeability of vacuum $\mu_0 = 4\times10^{-7}$ $T^2 \cdot m^3/J$
$= 12.566370614\times10^{-7}$ $T^2 \cdot m^3/J$

Permittivity of vacuum $\varepsilon_0 = 1/(\mu_0 \cdot c^2)$
$= 8.854187817\times10^{-12}$ $C^2/J \cdot m$

Electron rest mass $m_e = 9.1093897\times10^{-31}$ kg

Proton rest mass $m_p = 1.6726231\times10^{-27}$ kg

Neutron rest mass $m_n = 1.6749286\times10^{-27}$ kg

Etalon Formulas

Two parameters completely specify an etalon: the free spectral range (*FSR*) and the finesse ($\Im$). The *FSR* is the spacing (usually given in frequency) between transmission peaks. The finesse is the ratio of the free spectral range to the full width at half maximum (*FWHM*) of the transmission peak and is directly related to the reflectivity of the surface *R*.

$$FSR = \frac{c}{2nl} \qquad \Im = \frac{FSR}{FWHM} = \frac{\pi\sqrt{R}}{1-R}$$

c is the speed of light, *n* is the index of refraction of the etalon, and *L* is the thickness of the etalon.

At high finesse values (where *R* is very close to 100% or 1),

$$R \approx 1 - \frac{\pi}{\Im}$$

Finesse	Reflectivity
2	24%
4	47%
6	60%
8	68%
10	73%
15	81%
20	85%

Wave Vector, Frequency, Wavelength & Wavenumbers

$$k = \frac{2\pi}{\lambda} = \frac{2\pi n}{\lambda_0} = \frac{2\pi n\nu}{c} = \frac{n\omega}{c} \qquad \lambda = \frac{c}{n\nu} = \frac{\lambda_0}{n} = \frac{2\pi}{k} = \frac{2\pi c}{n\omega}$$

$$\nu = \frac{c}{\lambda_0} = \frac{c}{n\lambda} = \frac{kc}{2\pi n} = \frac{\omega}{2\pi} \qquad \Delta\lambda = \frac{c\,\Delta\nu}{\nu^2} = \frac{\lambda^2\Delta\nu}{c}$$

k = wave vector
ν = frequency
$\omega = 2\pi\nu$ = angular frequency
λ = wavelength
λ_0 = wavelength in vacuum
n = refractive index

An easy number to remember is a 1-pm linewidth is approximately 125 MHz at 1550 nm.

$$\text{Wavenumber (cm}^{-1}) = \frac{10^7}{\lambda\text{ (nm)}}$$

$$\text{Electron Volts (eV)} = \frac{1242}{\lambda\text{ (nm)}}$$

Wavelength (in vacuum), nm	Frequency, THz	Electron Volts, eV	Wavenumber, cm^{-1}
1561.42	192.00	0.80	6404.43
1550	193.41	0.80	6451.61
1320	227.12	0.94	7575.76
1064	281.76	1.17	9398.50
980	305.91	1.27	10204.08
780	384.35	1.59	12820.51
632.8	473.76	1.96	15802.78
350	856.55	3.55	28571.43

International System of Units (SI) Prefixes

Factor	Name	Symbol
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{-2}	centi	c
10^{-3}	mili	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

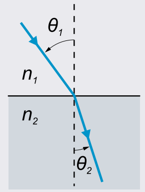
Common Material Properties

Material	Refractive Index, *n*	Δ*FSR**, MHz	Thermal Expansion Coefficient *α*, ppm/°C	Thermo-Optic Coefficient *β* or $\partial n/\partial T$, ppm/°C
Air	1.000	0.0	0.0	1.0
Fused Silica	1.444	13.1	0.55	6.57
Silicon	3.477	198.1	3.24	160
LASFN9	1.813	9.4	7.4	1.3

**Change in FSR due to dispersive effects as measured from 1510 to 1570 nm for a 50-GHz etalon*

Snell's Law

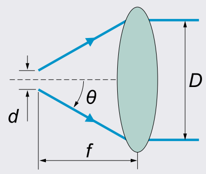
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



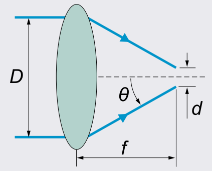
Numerical Aperture

$$f/\# = \frac{f}{D} \approx \frac{1}{2NA} \qquad NA = n\sin\theta$$

Collimating



Focusing



Reflection Air / Material

$$R = \left(\frac{n-1}{n+1}\right)^2 \text{ at AOI=0°}$$

Where n – refractive index,
AOI – Angle of Incidence.

Phase Matching Types of Nonlinear Crystals

Negative crystals ($n_o>n_e$)

Type 1 $k_{o1}+k_{o2}=k_{e3}(\theta)$
or "ooe interaction"

Type 2 $k_{e1}(\theta)+k_{o2}=k_{e3}(\theta)$
or "eoe interaction"

Type 2 $k_{o1}+k_{e2}(\theta)=k_e3(\theta)$
or "oee interaction"

Positive crystals ($n_e>n_o$)

Type 1 $k_{e1}(\theta)+k_{e2}(\theta)=k_{o3}$
or "eeo interaction"

Type 2 $k_{o1}+k_{e2}(\theta)=k_{o3}$
or "oeo interaction"

Type 2 $k_{e1}(\theta)+k_{o2}=k_{o3}$
or "eoo interaction"

Whereas k-wave propagation vector ($k=2\pi n/\lambda$); θ – phase matching angle in the crystal; o – ordinary polarization, e – extraordinary polarization; 1, 2, 3 indices – corresponds to wave vectors with longest (1), mid (2) and shortest (3) wavelengths.

Brewster's Angle

The angle where only *s*-polarized light is reflected

$$\theta_{Brewster} = \arctan\left(\frac{n_{transmitted\ medium}}{n_{incident\ medium}}\right)$$

Gausian Beam

A Gaussian beam spreads as follows,

$$\omega^2(x) = \omega_0^2\left[1+\left(\frac{\lambda x}{\pi\omega_0^2}\right)^2\right]$$

where $\omega(x)$ is the $1/e^2$ radius, λ is the wavelength, and x is the distance from the beam waist ω_0 where x=0.

A Rule of Thumb for Choosing a Lens

$$f = \frac{dD\pi}{4\lambda}$$

where *f* is the lens focal length, *d* is the beam diameter at the focus, *D* is the $1/e^2$ diameter of the collimated beam.

Nonlinear Crystal Thickness Limited by Group Velocity Mismatch (GVM)

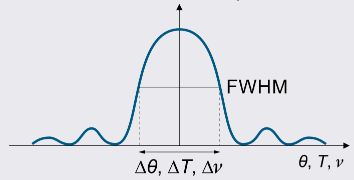
$$L = \frac{t}{GVM} \qquad GVM = \frac{1}{u_1} - \frac{1}{u_2}$$

$$u = \frac{c}{n(\lambda)}\left[1+\frac{\lambda}{n(\lambda)}\frac{\partial n(\lambda)}{\partial\lambda}\right]$$

Whereas *t* – pulse duration, *c* – speed of the light, *n* – refractive index, *λ* – wavelength.

Nonlinear Crystal acceptances

Nonlinear Crystal acceptances – Angular $\Delta\theta$, Temperature ΔT, Spectral $\Delta\nu$ – corresponding bandwidths at Full Width of Half Maximum (FWHM) of conversion efficiency.



Total Internal Reflection Angle

$$\theta_{TIR} > \arcsin\left(\frac{n_{transmitted\ medium}}{n_{incident\ medium}}\right)$$

where $n_{transmitted\ medium} < n_{incident\ medium}$ is required for total internal reflection.

Scaling Law for Laser Radiation Damage

$$E = E_1\sqrt{\frac{t}{t_1}}$$

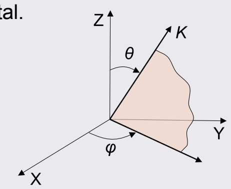
where E [J/cm²] is the damage threshold, t is the pulse duration, E_1 and t_1 are the reference damage threshold and pulse duration.

Non Critical Phase Matching

NCPM – when crystal phase matching angle equals 90° (θ = 90°). NCPM is achieved at special temperatures and/or wavelengths.

Uniaxial Crystals Refractivity

Polar coordinate system for description of refractive properties of uniaxial crystal.



Whereas K – light propagation vector at phase matching conditions, Z – optical axis of crystal, *θ* – phase matching angle (or cut angle), *φ* – azimuthal angle.

Birefrigency angle or Walk-off

$$\rho(\theta) = \pm\arctan\left[\left(\frac{n_o}{n_e}\right)^2 \tan(\theta)\right] \pm \theta$$

Upper signs refer to negative crystal ($n_o>n_e$) and the lower signs refer to positive one ($n_e>n_o$).

Beam displacement because of walk–off:

Δ = L tan (ρ)

Whereas L – crystal length, ρ – walk-off angle.