Useful Formulas & Constants

Physical Constants

Planck’s constant \( h = 6.6260755 \times 10^{-34} \text{ J} \cdot \text{s} = 4.5 \times 10^{-15} \text{ eV} \cdot \text{s} \)

Dirac’s constant \( \hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34} \text{ J} \cdot \text{s} = 8.62 \times 10^{-27} \text{ erg} \cdot \text{s} \)

Boltzmann’s constant \( k_B = 1.380 \times 10^{-23} \text{ J/K} = 8.62 \times 10^{-16} \text{ erg/K} \)

\( k_B T \) at room temperature

- 0.36 meV at liquid-helium temperature (4.2 K)
- 6.7 meV at liquid-nitrogen temperature (77 K)

Velocity of light in vacuum \( c = 2.99792458 \times 10^8 \text{ m/s} \)

Electron charge \( e = 1.602 \times 10^{-19} \text{ coulombs} \)

Avogadro number \( N_A = 6.0221367 \times 10^{23} \text{ particles/mol} \)

Permeability of vacuum \( \mu_0 = 4 \times 10^{-7} \text{ T}^2 \cdot \text{m}^3/\text{J} \)

Permittivity of vacuum \( \varepsilon_0 = \frac{1}{\mu_0 c^2} = 8.854187817 \times 10^{-12} \text{ C}^2/\text{J} \cdot \text{m} \)

Electronic rest mass \( m_e = 9.1093897 \times 10^{-31} \text{ kg} \)

Proton rest mass \( m_p = 1.6726231 \times 10^{-27} \text{ kg} \)

Neutron rest mass \( m_n = 1.6749286 \times 10^{-27} \text{ kg} \)

International System of Units (SI) Prefixes

Wave Vector, Frequency, Wavelength & Wavenumbers

\[ k = \frac{2\pi}{\lambda} = \frac{2\pi n \nu}{c} = \frac{2\pi n \nu}{n \omega} = \frac{2\pi n v}{\lambda} \]

\[ \nu = \frac{c}{\lambda_0} = \frac{c}{n\lambda} = \frac{kc}{2\pi n} = \frac{\omega}{2\pi} \]

\[ \Delta \lambda = \frac{c \Delta \nu}{n^2} = \frac{\lambda^2 \Delta \nu}{c} \]

At high finesse values (where \( R \) is very close to 100% or 1),

\[ R = 1 - \frac{\pi}{3} \]

Common Material Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Refractive Index, ( n )</th>
<th>( \Delta \text{FSR}^* ), MHz</th>
<th>Thermal Expansion Coefficient ( \alpha ), ppm/°C</th>
<th>Thermo-Optic Coefficient ( \beta ) or ( \partial n / \partial T ), ppm/°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.000</td>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Fused Silica</td>
<td>1.444</td>
<td>13.1</td>
<td>0.55</td>
<td>6.57</td>
</tr>
<tr>
<td>Silicon</td>
<td>3.477</td>
<td>198.1</td>
<td>3.24</td>
<td>160</td>
</tr>
<tr>
<td>LASFN9</td>
<td>1.813</td>
<td>9.4</td>
<td>7.4</td>
<td>1.3</td>
</tr>
</tbody>
</table>

*Change in FSR due to dispersive effects as measured from 1510 to 1570 nm for a 50-GHz etalon

Snell’s Law

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### Numerical Aperture

\[ f / \# = \frac{f}{D} = \frac{1}{2NA} \quad NA = n \sin \theta \]

### Brewster’s Angle

The angle where only s-polarized light is reflected

\[ \theta_{\text{Brewster}} = \arctan \left( \frac{n_{\text{transmitted medium}}}{n_{\text{incident medium}}} \right) \]

Where \( n_{\text{transmitted medium}} < n_{\text{incident medium}} \) is required for total internal reflection.

### Total Internal Reflection Angle

\[ \theta_{\text{TR}} > \arcsin \left( \frac{n_{\text{transmitted medium}}}{n_{\text{incident medium}}} \right) \]

### Gaussian Beam

A Gaussian beam spreads as follows,

\[ \omega^2(x) = \omega_0^2 \left( 1 + \left( \frac{\lambda x}{\pi \omega_0^2} \right)^2 \right) \]

where \( \omega(x) \) is the \( 1/e^2 \) radius, \( \lambda \) is the wavelength, and \( x \) is the distance from the beam waist \( \omega_0 \) where \( x = 0 \).

### Scaling Law for Laser Radiation Damage

\[ E = E_1 \sqrt{\frac{t}{t_1}} \]

where \( E \) [J/cm²] is the damage threshold, \( t \) is the pulse duration, \( E_1 \) and \( t_1 \) are the reference damage threshold and pulse duration.

### Phase Matching Types of Nonlinear Crystals

**Negative crystals** (\( n_o > n_e \))

Type 1  \( k_{11}(\theta)+k_{22}(\theta)=k_{33}(\theta) \)  or “ooe interaction”

Type 2  \( k_{21}(\theta)+k_{33}(\theta)=k_{11}(\theta) \)  or “eoe interaction”

Type 2  \( k_{31}(\theta)+k_{12}(\theta)=k_{23}(\theta) \)  or “eoo interaction”

**Positive crystals** (\( n_e > n_o \))

Type 1  \( k_{31}(\theta)+k_{22}(\theta)=k_{13}(\theta) \)  or “ooe interaction”

Type 2  \( k_{32}(\theta)+k_{12}(\theta)=k_{23}(\theta) \)  or “eoe interaction”

Type 2  \( k_{11}(\theta)+k_{22}(\theta)=k_{33}(\theta) \)  or “eoo interaction”

### Non Critical Phase Matching (NCPM)

\[ \text{NCPM} \rightarrow \text{when crystal phase matching angle equals } 90^\circ (\theta = 90^\circ). \text{NCPM is achieved at special temperatures and/or wavelengths.} \]

### Uniaxial Crystals Refractivity

Polar coordinate system for description of refractive properties of uniaxial crystal.

- \( K \) – light propagation vector at phase matching conditions,
- \( Z \) – optical axis of crystal,
- \( \theta \) – phase matching angle (or cut angle),
- \( \phi \) – azimuthal angle.

### Nonlinear Crystal Thickness Limited by Group Velocity Mismatch (GVM)

\[ L = \frac{t}{GVM} \quad \text{GVM} = \frac{1}{u_1} - \frac{1}{u_2} \]

\[ u = \frac{c}{n(\lambda)} \left[ 1 + \frac{\lambda}{n(\lambda)} \frac{dn}{d\lambda} \right] \]

Whereas \( t \) – pulse duration,
\( c \) – speed of the light,
\( n \) – refractive index,
\( \lambda \) – wavelength.

### Preferred Crystal Cut Angle

\[ \rho \text{ or } \delta = \pm \arctan \left( \left( \frac{\alpha}{\alpha} \right)^2 \tan(\theta) \right) \pm \theta \]

Upper signs refer to negative crystal (\( n_o > n_e \)) and the lower signs refer to positive one (\( n_e > n_o \)).

### Birefrigency angle or Walk-off

\[ \Delta = L \tan(\rho) \]

Whereas \( L \) – crystal length,
\( \rho \) – walk-off angle.